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— Theoretical Atomic Physics —

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Summer Term

Home Work (1)

Task 1: The Angular Momentum Operator

The angular momentum operator is defined as

$$\boldsymbol{l} = \boldsymbol{r} \times \boldsymbol{p} = -i\hbar \left(\boldsymbol{r} \times \nabla\right) . \tag{1}$$

Prove that the Cartesian components can be written as

$$l_x = i\hbar \left(\sin\varphi \frac{\partial}{\partial\vartheta} + \cot\vartheta \cos\varphi \frac{\partial}{\partial\varphi} \right) , \tag{2}$$

$$l_y = i\hbar \left(-\cos\varphi \frac{\partial}{\partial\vartheta} + \cot\vartheta \sin\varphi \frac{\partial}{\partial\varphi} \right) , \tag{3}$$

$$l_z = -i\hbar \frac{\partial}{\partial\varphi} . \tag{4}$$

Task 2: Commutation Relations of the Angular Momentum Operator

Prove the following commutation relations

a) $\left[H, \boldsymbol{l}^2\right] = 0$, $[H, l_i] = 0$,

b) $[l_i, l_j] = i\hbar\epsilon_{ijk} l_k$

where the Hamiltonian is given by the Schrödinger Hamiltonian with a spherically symmetric potential.

Task 3: Creation of Holes in the Dirac Sea

How much energy is required to create an electron-position pair in the field of a calcium nucleus ($Z = 20$) if the electron is captured into the $1s$ ground state of the ion.

Hint: Use the non-relativistic formula for the bound-state energies, $E_n = -Z^2/2n^2$[a.u.].

Task 4: Relativistic Spin-Orbit Operator

Evaluate the (4×4) representation of the relativistic spin-orbit operator

$$k = \alpha_0 \left(\boldsymbol{l} \cdot \boldsymbol{\sigma}_D + \hbar \cdot \mathbf{1} \right) \tag{5}$$

$$\sigma_{D_i} = \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} \tag{6}$$

Task 5: Muonic Hydrogen

The muon is a fermion, much like an electron, but with mass $m_\mu = 105.7\,\mathrm{MeV/c^2}$ and same charge. In a hydrogen atom, the muon can be captured and therefore replace the electron, to obtain *muonic hydrogen*.

a) What is the ground state binding energy of the muon?

b) What chemical element does muonic lithium, where only one electron is replaced by a muon, resemble most?