

Home Work (4)

Task 16: Isotope shift in lithium

Estimate the normal mass shift of the $2s - 2p$ transition in Lithium for the two isotopes ${}^6\text{Li}$ and ${}^7\text{Li}$. Use the experimentally available transition energy, for your estimate.

Task 17: Spin-orbit coupling

Calculate the expectation value of the spin-orbit coupling Hamiltonian

$$H_{\text{LS}} = \frac{\mu_B}{c^2} \frac{1}{r} \frac{\partial V}{\partial r} \mathbf{L} \cdot \mathbf{S}$$

for the eigenstates of a hydrogen-like atom.

Hint: You can use

$$\langle r^{-1} \rangle = \frac{1}{n^2} \qquad \langle r^{-2} \rangle = \frac{1}{(l + \frac{1}{2}) n^3 a^2}$$

where $a = a_0/Z$, as well as Kramer's relation:

$$\frac{a^2 s}{4} \left((2l + 1)^2 - s^2 \right) \langle r^{s-2} \rangle - a(2s + 1) \langle r^{s-1} \rangle + \frac{s + 1}{n^2} \langle r^s \rangle = 0$$

Task 19: Angular Momentum coupling (2)

a) Prove that only three terms (${}^1\text{S}$, ${}^1\text{D}$, ${}^3\text{P}$) are allowed for the ground-state configuration $1s^2 2s^2 2p^2$ of a carbon-like system. Work in LS-coupling.

b) Find all terms, which are possible for the electronic configuration d^2 .

c) Couple the two valence shells to the electronic configuration $p^2 d^2$

Task 18: Relativistic angular momentum coupling

List the possible terms ($n_1 j_1, n_2 j_2, \dots$) and total angular momentum J in jj-coupling for the following electronic configurations:

- $1s^2$
- $1s2s$
- $1s^2 2s^2 2p$
- $1s^2 2s^2 2p^5$
- $ns n'p$
- $np n'd$