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— Computational Quantum Physics —

30 April 2018
Sommersemester

Exercise 2

Task 1: Unitary transformation of a Hermitian operator (2 points)

Suppose A is a hermitian operator, U denotes a unitary operator and $A' = UAU^\dagger$. Show that

a) A' is Hermitian.

b) A' has the same eigenvalues as A.

Task 2: Entanglement (2 points)

Prove that the entangled Bell state $|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ cannot be represented in the form $|\psi\rangle = |a\rangle\,|b\rangle$ where $|a\rangle$ and $|b\rangle$ are single-qubit states.

Task 3: Inequalities and orthonormalization (8 points)

Consider the three-qubit state $|\phi\rangle = |+++\rangle$, where $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, and the Greenberger–Horne–Zeilinger state

$$|\psi\rangle = \left|\mathrm{GHZ}^{(3)}\right\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle).$$

a) Show that these states obey the Cauchy–Schwarz and triangle inequalities:

$$|\langle\phi|\psi\rangle|^2 \leq \langle\phi|\phi\rangle\,\langle\psi|\psi\rangle\,, \qquad \sqrt{\langle\phi+\psi|\phi+\psi\rangle} \leq \sqrt{\langle\phi|\phi\rangle} + \sqrt{\langle\psi|\psi\rangle}.$$

b) Orthomormalize these states by means of the Gram–Schmidt procedure.

c) What would be an appropriate data structure to implement a n-qubit quantum register (pure states only, no density matrix)? How many values need to be stored?

d) Implement the n-qubit states $\left|\phi^{(n)}\right\rangle = |+\cdots+\rangle$ and $\left|\psi^{(n)}\right\rangle = \left|\mathrm{GHZ}^{(n)}\right\rangle$. Check the inequalities from a) and perform the orthonormalization from b) for the three-qubit states $|\phi\rangle$ and $|\psi\rangle$ as well as for the analogous four- and five-qubit states.