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— Computational Quantum Physics —

28 May 2018
Sommersemester

Exercise 3

Task 1: Measurement of a three-qubit state (2 points)

A three-qubit system is in the state

$$|\psi\rangle = \frac{\sqrt{2}+\mathrm{i}}{\sqrt{20}}\,|000\rangle + \frac{1}{\sqrt{2}}\,|001\rangle + \frac{1}{\sqrt{10}}\,|011\rangle + \frac{\mathrm{i}}{2}\,|111\rangle\,.$$

a) Is this state normalized? What is the probability of finding the system in the state $|000\rangle$ if all three qubits are measured?

b) What is the probability that a measurement of only the first qubit yields $|0\rangle$? What is the post-measurement state of the system in this case?

Task 2: Projectors of $\boldsymbol{v}\cdot\boldsymbol{\sigma}$ (6 points)

Let $\boldsymbol{v} = (v_1, v_2, v_3)$ be a real, three-dimensional unit vector and $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ the vector of Pauli matrices.

a) Show that the operator $\boldsymbol{v}\cdot\boldsymbol{\sigma} = v_1\sigma_1 + v_2\sigma_2 + v_3\sigma_3$ has the eigenvalues ± 1 and that the projection operators upon the corresponding eigenspaces are given by $P_\pm = \frac{I+\boldsymbol{v}\cdot\boldsymbol{\sigma}}{2}$, where I is the identity matrix.

b) Apply the projector $P_\pm$ to calculate the probabilities that a measurement of the operator $\boldsymbol{v}\cdot\boldsymbol{\sigma}$ on the state $|0\rangle$ yields the results ± 1. What is the state of the qubit after measuring $+1$?

Task 3: Single-qubit rotations (4 points)

Let $\boldsymbol{v} = (v_1, v_2, v_3)$ be a real, three-dimensional unit vector, $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ the vector of Pauli matrices, and θ a real number. Prove that

$$\exp(\mathrm{i}\theta\boldsymbol{v}\cdot\boldsymbol{\sigma}) = \cos\theta I + \mathrm{i}\sin\theta\boldsymbol{v}\cdot\boldsymbol{\sigma}.$$