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— Computational Quantum Physics —

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Summer Term

Homework I

Task 1: Commutation Relations of the Angular Momentum Operator

Prove the following commutation relations
a) $\left[H, \boldsymbol{l}^2\right] = 0$, $[H, l_i] = 0$,
b) $[l_i, l_j] = i\hbar\epsilon_{ijk} l_k$
where the Hamiltonian is given by the Schrödinger Hamiltonian with a spherically symmetric potential.

Task 2: The Angular Momentum Operator

The angular momentum operator is defined as

$$\boldsymbol{l} = \boldsymbol{r} \times \boldsymbol{p} = -i\hbar \left(\boldsymbol{r} \times \nabla\right) . \tag{1}$$

Prove that the Cartesian components can be written as

$$l_x = i\hbar \left(\sin\varphi \frac{\partial}{\partial\vartheta} + \cot\vartheta \cos\varphi \frac{\partial}{\partial\varphi} \right) , \tag{2}$$

$$l_y = i\hbar \left(-\cos\varphi \frac{\partial}{\partial\vartheta} + \cot\vartheta \sin\varphi \frac{\partial}{\partial\varphi} \right) , \tag{3}$$

$$l_z = -i\hbar \frac{\partial}{\partial\varphi} . \tag{4}$$

Task 3: Computational task with the usage JAC

For hydrogen-like gold ($Z = 79$), determine the energies of the $3d$ level for the relativistic case in eV. Compare the obtained results with non-relativistic ones (Schrödinger equation).