Problems

1) Prove that Green's function:

$$G(E,\boldsymbol{r},\boldsymbol{r}') = \sum_n \frac{\varphi_n(r)\varphi_n^*(r')}{E - E_n}$$

where summation runs over the complete spectrum of the system and where $(\varphi_n(r), E_n)$ are the eigenfunctions and eigenenergies of some Hamiltonian *H*, is really solution of the equation:

$$(E - \hat{H})G(E,\boldsymbol{r},\boldsymbol{r}') = \delta(\boldsymbol{r} - \boldsymbol{r}')$$

2) In our analysis of the potential scattering we made use of the <u>free-particle</u> Green's function and the corresponding Green's operator:

$$\hat{G}_0 = \frac{1}{E - \hat{H}_0}$$

Write the Green's operator $\hat{G}$ of the particle, moving into some potential *V*. Show how this operator is expressed in terms of $\hat{G}_0$.

Hint: Use the operator relation: $\frac{1}{\hat{A}} - \frac{1}{\hat{B}} = \frac{1}{\hat{A}}(\hat{B} - \hat{A})\frac{1}{\hat{B}}$

Problems

3) Prove that scattered wave in the asymptotics of wave-function $\psi_{\boldsymbol{k}}{}^{(+)} = e^{i\boldsymbol{kr}} + f(\boldsymbol{k}', \boldsymbol{k})\frac{e^{ikr}}{r}$ really describes the outgoing wave.

4) Find asymptotics of the wave-function: $\psi_k{}^{(-)}(\boldsymbol{r}) = e^{i\boldsymbol{kr}} + \int G_0{}^{(-)}(E, \boldsymbol{r}, \boldsymbol{r}')V(\boldsymbol{r}')\psi_k(\boldsymbol{r}')d\boldsymbol{r}'$