

Problems (for 18 December)

1. Prove that in the limit $|V| \ll E/(kd)$ the eikonal scattering amplitude becomes the amplitude which was obtained within the first Born approximation.
2. Calculate the eikonal for high-energy scattering by the potential:

$$V(r) = \begin{cases} V_0, & r \leq d \\ 0, & r > d \end{cases}$$

3. Calculate the total scattering cross section in terms of the eikonal $\chi(\mathbf{b})$. Prove that the optical theorem is valid in the eikonal approximation.
4. For the 1-D wave-packet:

$$\Psi(x, t = 0) = e^{ikx} \cdot \frac{1}{b^{1/2} \pi^{1/4}} \exp\left(-\frac{(x - x_0)^2}{2b^2}\right)$$

find the variance of the coordinate $D_x = \langle x^2 \rangle - \langle x \rangle^2$ and of the momentum $D_p = \langle p^2 \rangle - \langle p \rangle^2$