Exercises: Tutorial 05.11.2015

1. Calculate de Brogile wavelength of the electron in the n=1,2,3 states of hydrogen atom. Present results either in atomic units (Bohr radius) or nanometers. Compare these results with the wavelengths of light emitted in n=2 → n=1, n=3 → n=1 transitions.

2. Derive the Heisenberg uncertainty relation $[x, p_x] = i\hbar$. Use explicit form of linear momentum operator in Cartesian coordinates.

3. Prove the commutation relations for the orbital angular momentum operator:

$$[\hat{l}_x, \hat{l}_y] = i\hat{l}_z, \qquad [\hat{l}_y, \hat{l}_z] = i\hat{l}_x, \qquad [\hat{l}_z, \hat{l}_x] = i\hat{l}_y$$

   Use the explicit form of the components of this operator in Cartesian coordinates.

4. Derive the basic relations for the lowering and rising operators:

$$[\hat{j}^2, \hat{j}_\pm] = 0, \qquad [\hat{j}_z, \hat{j}_\pm] = \pm\hat{j}_\pm, \qquad [\hat{j}_+, \hat{j}_-] = 2\hat{j}_z$$

5. We have discussed during the lecture that lowering/rising operators act as $\hat{j}_\pm|jm\rangle = C_\pm|j\ m \pm 1\rangle$. Find the coefficient $C_\pm$.

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