Exercises: Tutorial 20.11.2015 (part 1)

1. Derive the explicit form of the orbital angular momentum operator l_x in spherical coordinates. I.e. prove the expression:

$$\hat{l}_x = i\hbar\left(\sin\varphi \frac{\partial}{\partial\theta} + \cot\theta\cos\varphi \frac{\partial}{\partial\varphi} \right),$$

2. Prove the commutation relations for the spin operators:

$$[\hat{s}_x, \hat{s}_y] = i\hat{s}_z, [\hat{s}_y, \hat{s}_z] = i\hat{s}_x, [\hat{s}_z, \hat{s}_x] = i\hat{s}_y$$

3. Prove that for the system in the quantum state |j m_j> where m_j is the projection of the momentum on the z-axis, the expectation values of operators j_x and j_y are zero:

$$\left\langle jm_j \middle| \hat{j}_x \middle| jm_j \right\rangle = \left\langle jm_j \middle| \hat{j}_y \middle| jm_j \right\rangle = 0$$

HI Jena
Helmholtz Institute Jena

Exercises: Tutorial 20.11.2015 (part 2)

4. Prove that sum of two angular momentum operators is also angular momentum operator:

$$\hat{J} = \hat{j}_1 + \hat{j}_2$$

5. Prove that operators $\hat{J}^2, \hat{J}_z, \hat{j}_1^2, \hat{j}_2^2$ commute with each other.

6. Find the minimal value of the (total) angular momentum from the expression:

$$\sum_{J_{\min}}^{j_1+j_2} (2J+1) = (2j_1+1)(2j_2+1)$$