Exercises: Tutorial 15.01.2016 (part 1)

1. Write down the state $\widetilde{\left|s=\frac{1}{2}\quad m_s=\frac{1}{2}\right\rangle}$ which describes the "spin up" (along $\tilde{z}$ the axis) in the system rotated by angle $\theta = 90$ deg with respect to un-rotated system.

2. Prove the orthogonality property of Wigner D functions:

$$\int d\Omega\ D^{*\,j_1}_{m_1' m_1}(\varphi,\theta,\chi) D^{j_2}_{m_2' m_2}(\varphi,\theta,\chi) = \frac{8\pi^2}{2j_1+1}\ \delta_{j_1 j_2}\ \delta_{m_1 m_2}\delta_{m_1' m_2'}$$

Exercises: Tutorial 15.01.2016 (part 2)

3. Prove that the scalar (dot) product of two vectors **A** and **B** reads as

$$\boldsymbol{A} \cdot \boldsymbol{B} = \sum_{q=-1}^{+1} A_q^* \, B_q$$

where A_q and B_q are spherical components of the vectors.

4. Calculate the integral $\int [Y_{2m}(\theta, \varphi)]^3 \, d\Omega$

5. Consider a particle with the orbital momentum *l*=2 and its projection *m*=1 onto the axis z of the (non-rotated) system. Find the probability $W(m')$ that a particle would have the projection *m'* of the momentum *l* onto the axis $\tilde{z}$ which is rotated by the angle 60 deg with respect to the z.