Exercises: Tutorial 29.01.2016

1. Quite often in the literature one uses alternative (mathematically equivalent) definition of the tensor operator of rank k:

$$\left[\hat{J}_z, \hat{T}_{kq}\right] = q\,\hat{T}_{kq}, \qquad \left[\hat{J}_{\pm}, \hat{T}_{kq}\right] = \sqrt{k(k+1) - q(1 \pm 1)}\,\hat{T}_{k\,q\pm1}$$

Prove these relations.

2. Electric quadrupole (E2) γ transitions in nuclei are described by the quadrupole operator $\hat{Q}_{2q}$ which is constructed as a irreducible spherical tensor of rank 2. Based on the analysis of the matrix element of this operator, find selection rules for the E2 transition.

3. Find the reduced matrix element of the angular momentum operator $\langle j \| \hat{J} \| j' \rangle$

4. Calculate the matrix element $\langle j_1\, j_2 : J | \hat{\sigma}_1 \cdot \hat{\sigma}_2 | j_1'\, j_2' : J' \rangle$ where $j_i = l_i + s_i$

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